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Estimating Hop Counts in Position Based Routing Schemes for Ad Hoc Networks *

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Abstract. The recent availability of small, inexpensive low power GPS receivers and techniques for finding relative coordinates based on signal strengths, and the need for the design of power efficient and scalable networks, provided justification for applying position based routing methods in ad hoc networks. A number of such algorithms were developed recently. They are all based on three greedy schemes, applied when the forwarding node is able to advance the message toward destination. In this paper we show that the hop count, that is the number of transmissions needed to route a message from a source node to a destination node can be estimated reasonably accurately (in random unit graphs with uniform traffic), with less than 10%, 5% and 7% error for directional (compass), distance (greedy) and progress (MFR) based schemes, respectively, for 100 nodes with average degrees between 5 and 14, without experiments. Our results are derived from statistical observations regarding expected position of forwarding neighbor.

Keywords: ad hoc networks, routing, GPS, greedy algorithms

Introduction

Ad hoc networks consist of wireless hosts that communicate with each other in the absence of a fixed infrastructure. They have potential applications in disaster relief, conference and battlefield environments, and received significant attention in recent years. Sensor networks are a class of wireless ad hoc networks. Wireless networks of sensors are likely to be widely deployed in the near future because they greatly extend our ability to monitor and control the physical environment from remote locations and improve our accuracy of information obtained via collaboration among sensor nodes and online information processing at those nodes. Networking these sensors (empowering them with the ability to coordinate amongst themselves on a larger sensing task) will revolutionize information gathering and processing in many situations. Other contexts include rooftop networks, static networks with nodes placed on top of buildings, to be used when wired networks fail.

In an ad hoc network, a message sent by a node reaches all its neighboring nodes that are located at distances up to the transmission radius. Because of the limited trans-

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mission radius, the routes between nodes are normally created through several hops in such multi-hop wireless networks. In the widely accepted *unit* graph model, two nodes A and B in the network are neighbors if the distance between them is at most R, where R is the transmission radius which is equal for all nodes in the network.

In this article we consider the routing task, in which a message is to be sent from a source node to a destination node in a given wireless network. The task of finding and maintaining routes in sensor and ad hoc networks is nontrivial since host mobility and changes in node activity cause frequent unpredictable topological changes. The destination node is known and addressed by means of its location. We assume that the position of destination is accurate. The problem of designing location update schemes to provide accurate destination information and enable efficient routing in mobile ad hoc networks appears to be more difficult than routing itself and will not be discussed here (a recent informative survey is given in [Stojmenovic, 6]).

The distance between neighboring nodes can be estimated on the basis of incoming signal strengths or time delays in direct communications. Relative coordinates of neighboring nodes can be obtained by exchanging such information between neighbors. Alternatively, the location of nodes may be available directly by communicating with a satellite (for outdoor networks), using GPS (Global Positioning System), if nodes are equipped with a small low power GPS receiver. The position based approach in routing becomes practical due to the rapidly developing software and hardware solutions for determining absolute or relative positions of nodes in indoor/outdoor ad hoc networks [Hightower and Borriello, 3].

The routing algorithms should perform well for wireless networks with an arbitrary number of nodes. Sensor and rooftop networks, for instance, have hundreds or thousands of nodes. While other characteristics of each algorithm are easily detected, *scalability* is sometimes judgmental, and/or dependent on the performance evaluation outcome. A scalable solution is the one that performs well in a large network. It has been experimentally confirmed [Li, 5; Stojmenovic, 6] that routing protocols that do not use geographic location in the routing decisions, such as *AODV, DSDV* or *DSR* (their recent survey is given in [Tseng et al., 10]) are not scalable. Therefore, it is likely that only position-based approaches provide satisfactory performance for large networks. Scalability is provided mainly by applying localized algorithms.

Localized algorithms are distributed in nature and resemble greedy algorithms, where simple local behavior achieves a desired global objective. In a *localized* routing algorithm, each node makes a decision to which neighbor to forward the message based solely on the location of itself, its neighboring nodes, and the destination. Such routing schemes are known as the *position-based schemes*. In the shortest (weighted) path based non-localized algorithms, each node maintains accurate topology of the whole network. In addition, since nodes change between active and sleep periods, the activity status for each node is also required. Although routing table (typical non-position) based solutions merely keep the best neighbor information on a route toward the destination, the communication overhead for maintenance of routing tables due to node mobility and topology changes is quadratic in network size (each change in edge or node status may trigger routing table modifications in large portion of the network). On the other hand, position based localized algorithms avoid that overhead, by requiring only accurate neighborhood information, and a rough idea on the position of the destination. For example, edge and node changes in one part of the network have no immediate impact on almost any route. Clearly, only localized algorithms provide scalable solutions, especially for networks with critical power constrained resources at nodes (e.g., sensor networks).

1. Greedy routing schemes

In a localized routing scheme, node S, currently holding the message, is aware only about the position of its neighbors within the transmission radius and destination D (indicated by black circles in figure 1).

Takagi and Kleinrock [9] proposed the first position based routing scheme, based on the notion of progress. Given a transmitting node S, the *progress* of a node A is defined as the projection onto the line connecting S and D. In the *MFR* (Most Forward within Radius) scheme [Takagi and Kleinrock, 9], the packet is forwarded to a neighbor whose progress is maximal, such as node M in figure 1.

Finn [2] proposed the greedy routing scheme based on geographic distance. S selects neighboring node G (see figure 1) that is closest to the destination among its neighbors. Only neighbors closer to the destination than S are considered. Otherwise there is a lack of advance, and method fails. A variant of this method is called *GEDIR* (GEographic DIstance Routing) scheme [Stojmenovic and Lin, 7]). In this variant, applied on other schemes as well, all neighbors are considered, and the message is dropped if the best choice for a current node is to return the message to the node the message came from (stoppage criterion indicating lack of advance).

In the *compass routing* method (referred also as the *directional* method) proposed by Kranakis et al. [4], the message m is forwarded to the neighbor A (see figure 1),

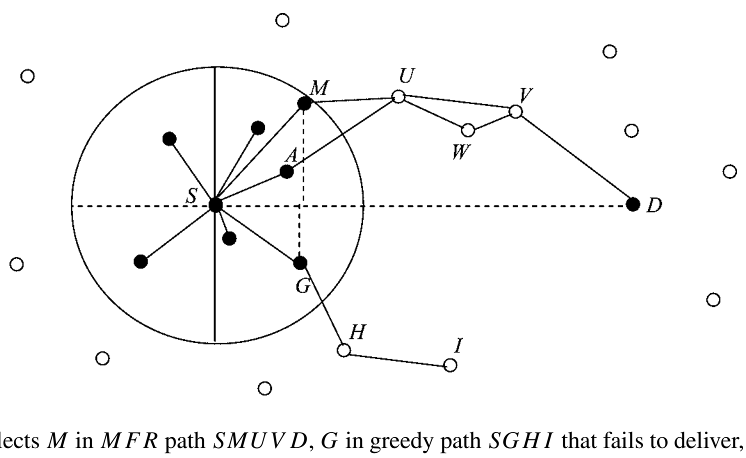

Figure 1. S selects M in MFR path $SMUVD$, G in greedy path $SGHI$ that fails to deliver, A in direction based path $SAUWVD$.

such that the direction SA is closest to the direction SD (that is, the angle $\angle ASD$ is minimized).

The *MFR* and *greedy*/*GEDIR* methods, in most cases, provide the same path to the destination, and are loop-free [Stojmenovic and Lin, 7]. The hop count for the *directional* method is somewhat higher than for the *greedy* scheme, while the success rate is similar. All methods have high delivery rates for dense graphs, and low delivery rates for sparse graph (about half the messages at average degrees below 4 are not delivered) [Stojmenovic and Lin, 7]. When successful, hop counts of greedy and *MFR* methods nearly match the performance of the shortest path algorithm. The *directional* method, and any other method that includes forwarding the message to a neighbor with closest direction, such as *DREAM* and *LAR,* are not loop-free (see [Stojmenovic and Lin, 7] for counterexample and references).

The performance of these schemes depends on the network density. Greedy schemes have a performance close to performance of optimal shortest path algorithm for dense graphs, but have low delivery rates for sparse graphs. Hop count was traditionally used to measure the energy requirement of a routing task, thus using a constant metric per hop.

In this paper, we study the hop count for the three basic position-based routing schemes. We shall derive formulas for their estimation that is very close to the data obtained by simulation [Stojmenovic and Lin, 7]. This will provide such estimation for larger number of nodes or larger densities, that is, beyond the data available experimentally.

2. Expected distance between two nodes in random unit graph

In this section we shall estimate the average distance between two nodes (e.g., source and destination) in a random unit graph with n nodes and average density (average number of neighbors of each node) k. Let the nodes be placed inside a square of with side lengths m. The transmission radius needed to achieve the desired node density can be estimated using the well known relation $k = \pi R^2/m^2(n-1)$ giving the expected number of nodes inside a circle centered at one of nodes. It is proportional to the ratio of areas of circle with radius R (transmission radius of the unit graph) and area of the square. This is the factor to be multiplied with the remaining number $n-1$ of nodes. Solving this equation yields $R = m\sqrt{k/(\pi(n-1))}$. Note that Li [5] provides more accurate estimation that accounts for border effects, but we believe that routing tasks do not reflect their impact, and will use this simplified formula.

Lemma 1. If two points are chosen at random in the interval [0, 1], their expected distance is $1/3$.

Proof. Let $x \leqslant y$ be two chosen points. All such pairs make a triangle in x-y coordinate plane. For each such pair, we consider the weight $y - x$ which is the distance between the two points. The expected distance is the average weight, and can be obtained by considering the double integral

$$\int_0^1 \left(\int_x^1 (y - x)\, \mathrm{d}y \right) \mathrm{d}x = \int_0^1 \left(\frac{1}{2} - x + \frac{x^2}{2} \right) \mathrm{d}x = 1/6.$$

The area of the triangle is $1/2$, thus the average distance is $1/6 : 1/2 = 1/3$. □

Lemma 2. If two points are chosen at random inside an s-dimensional square with side length m, the expected distance between them is $m\sqrt{s}/3$.

Proof. The expected distance along each dimension is $m/3$. Therefore the expected distance between them is $\sqrt{(m/3)^2 + \cdots + (m/3)^2} = m\sqrt{s}/3$. □

All the results in the rest of this paper assume uniform traffic and uniform random distribution of nodes inside a square.

3. Estimating hop count in directional method

We will first find the expected advance from source S to destination D in one hop. Interestingly, it will not depend on the distance $d = |SD|$ from source to destination, measured in local terms. We shall restrict our analysis to $s = 2$, that is, two-dimensional case.

Lemma 3. The expected position of forwarding neighbor in directional method in a unit graph with average density k and transmission radius R is a point at distance $R\sqrt{2}/2$ and at angle $\pi/(2k)$ with respect to direction SD.

Proof. Divide the region around S into k equal angular ranges, so that direction SD is bisector of one of these regions (see figure 2 where $k = 3$). Each such region is expected to contain one of k neighboring nodes. The one node A that is located in the region containing line SD is expected to make an angle $\pi/(2k)$ with respect to direction SD.

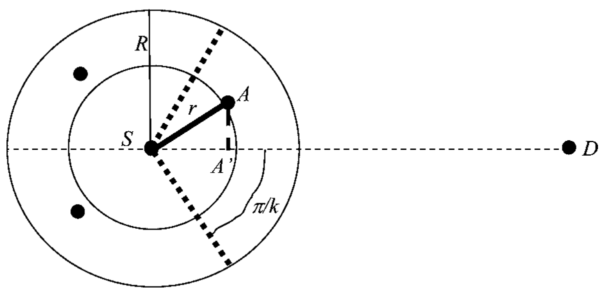

Figure 2. Expected advance in the directional based method.

Node A will be selected by the directional method. Next, we will find how far, on average, node A from S is. Let $r = |SA|$. The circle of radius r is expected to divide the circle of radius R into two areas of equal size, that is $\pi R^2 = 2\pi r^2$. That is, $r = R\sqrt{2}/2$. This determines the expected position A of forwarding node in one hop. □

Theorem 1. The expected hop count in directional based method is $\geqslant \sqrt{2}|SD|/(R\cos(\pi/(2k))$.

Proof. Let A' be the projection of point A on the direction SD. Clearly $|AD| \geqslant |A'D| = |SD| - |SA'|$. Since $|SA'| = R\sqrt{2}/2\cos(\pi/(2k))$, the number of expected hops is $\geqslant |SD|/|SA'| \geqslant \sqrt{2}|SD|/(R\cos(\pi/(2k))$. □

The error obtained by using theorem 1 to approximate hop count is below 13% for $n = 100$ nodes and densities k from 5 to 15, using data from [Stojmenovic and Lin, 7] for exact directional-based scheme hop count. As a more precise estimate, we investigated adding 0.5 to the lower bound obtained by theorem 1, and the estimated hop count is included in table 1. The error has been reduced to below 10%, obtained as an overestimate.

We also investigated another estimate that can be obtained by the following algorithm, which calculates new expected distance $|AD|$ as replacement to $|SD|$, and counts hops until the expected distance falls bellow transmission radius R.

Algorithm.
Input: n, k, m
$d = |SD| = m\sqrt{2}/3$
$R = m\sqrt{k/(\pi(n-1))}$
$d1 = |SA'| = R\sqrt{2}/2\cos(\pi/(2k))$
$Hop\_count = 0$
REPEAT
  $Hop\_count = Hop\_count + 1$
  $d = \sqrt{(d - d1)^2 + R^2\sin^2(\pi/2k)/2}$
UNTIL $d \leqslant R$
$Hop\_count = Hop\_count + d/R + 1$.

After exiting the loop, a correction is made in the hop count, by adding 1 for the fact that one more hop is needed to deliver the message to destination, and the fact that the remaining distance has impact on the hop count considered as a real number, reflecting an average case.

Table 1 compares data obtained by experiments [Stojmenovic and Lin, 7] with data obtained from theorem 1, its correction by adding 0.5 to the hop count, and above algorithm. The number of nodes is $n = 100$, the square has sides $m = 100$, the average distance is therefore 47.14. It can be observed that theorem 1 underestimates the hop count by at most 12% while the algorithm can overestimate by at most 16% or underes-

Table 1
Comparing experimental data with data from theorem 1 and algorithm.

Density k	5	6	7	8	9	10	11	12	13	14
R	12.68	13.89	15	16	17.01	17.93	18.81	19.64	20.44	21.22
SA'	8.53	9.49	10.34	11.1	11.85	12.52	13.16	13.77	14.35	14.91
Directional	5.92	5.55	5.13	4.63	4.55	4.17	4.06	3.72	3.6	3.34
Theorem 1	5.53	4.97	4.56	4.24	3.98	3.76	3.58	3.42	3.29	3.16
Theorem 1/Dir	0.93	0.9	0.89	0.91	0.87	0.9	0.88	0.92	0.91	0.95
Algorithm	6.69	5.19	5.3	4.48	4.59	4.53	4.42	4.32	3.23	3.16
Alg/Dir	1.13	0.93	1.03	0.97	1.01	1.09	1.09	1.16	0.9	0.95
$d/SA' + 0.5$	6.03	5.47	5.06	4.74	4.48	4.26	4.08	3.92	3.79	3.66
$(d/SA' + 0.5)$/Dir	1.02	0.99	0.99	1.02	0.98	1.02	1	1.05	1.05	1.1

Table 2
Estimating hop count for directional method for $n = 1000$ nodes.

Density k	100	5	6	7	8	9	10	11	12	13	14
R	17.85	3.99	4.37	4.72	5.05	5.36	5.64	5.92	6.16	6.47	6.68
SA'	12.62	2.68	2.99	3.26	3.5	3.73	3.94	4.14	4.36	4.52	4.69
$d/SA' + 0.5$	4.24	17.56	15.79	14.48	13.5	12.64	11.96	11.38	10.87	10.43	10.04

timate by at most 7%. The corrected theorem 1 appears to be most accurate, with error within 10% for all densities.

Table 2 presents estimated hop counts for 1000 nodes, for which no experimental data are currently available. The selected densities are 5–14 and 100.

4. Estimating hop count in MFR and greedy methods

In case of distance based greedy method, all points inside circle of radius R around source S that are equidistant from destination D lie on a circular segment centered at D. If k such circular segments are selected so that the circle centered at S with radius R is divided into k regions of equal areas, each of these areas will be expected to contain one neighbor of S. The packet will be forwarded to the region closest to destination, and expected position of forwarding node is on a circular segment that bounds region with area $1/(2k)$ of the circular area (that is, bisects closest of the k regions). The calculation involved, however, is sophisticated (though possible via numerical methods). We shall instead consider a simpler case of *MFR* method, since the performance of distance greedy method and *MFR* method were shown to be very close in hop counts and success rates [Stojmenovic and Lin, 7].

Consider therefore *MFR* method. The circle with radius R centered at S can be divided by lines orthogonal to direction SD into k regions with equal areas. Each of these regions is expected to contain one of k neighbors of S (see figure 3 for an illustration for $k = 5$). The region closest to D can be further subdivided into two subregions of equal area to find the estimated position A of selected neighbor. Thus the area of circular segment obtained by cutting the circle with a line orthogonal to SD should be $\pi R^2/(2k)$.

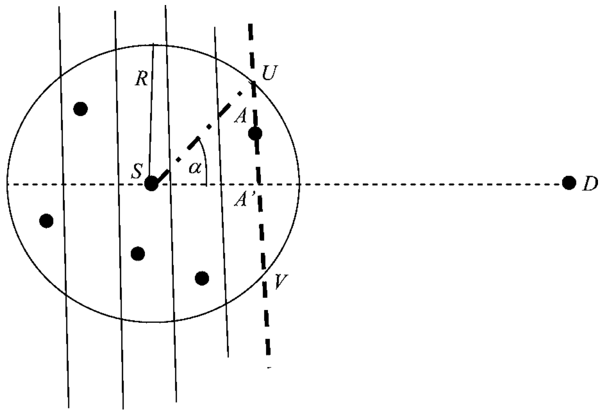

Figure 3. Expected advance in the *MFR* method.

Table 3
Estimating hop counts for *GEDIR* and *MFR* methods.

Density k	5	6	7	8	9	10	11	12	13	14
R	12.68	13.89	15	16	17.01	17.93	18.81	19.64	20.44	21.22
1_GEDIR	5.53	5.13	4.72	4.29	4.19	3.87	3.72	3.39	3.27	3.04
1_MFR	5.61	5.16	4.78	4.33	4.23	3.89	3.75	3.42	3.29	3.07
α	0.81	0.76	0.72	0.69	0.66	0.63	0.61	0.6	0.58	0.56
s/x	5.41	4.69	4.18	3.8	3.5	3.26	3.07	2.9	2.76	2.63
$(d/x)/MFR$	0.96	0.91	0.87	0.88	0.83	0.84	0.82	0.85	0.84	0.86
$d/x + 0.5$	5.91	5.19	4.68	4.3	4	3.76	3.57	3.4	3.26	3.13
$(d/x + 0.5)/MFR$	1.05	1.01	0.98	0.99	0.95	0.97	0.95	1	0.99	1.02
$(d/x + 0.5)/GEDIR$	1.07	1.01	0.99	1	0.95	0.97	0.96	1	1	1.03

Let U and V be the intersections of that line with the circle centered at S with radius R. Let $\alpha = \angle USD$. The area of the segment is then also $(\alpha/\pi)(\pi R^2) - R\sin(\alpha)R\cos(\alpha) = \alpha R^2 - R^2/2 \cdot \sin(2\alpha)$. Therefore $\pi R^2/(2k) = \alpha R^2 - R^2/2 \cdot \sin(2\alpha)$. Thus the equation for determining α is $\pi/(2k) = \alpha - \sin(2\alpha)/2$. It can be solved by numerical methods, e.g., bisection method. Per hop advance is $x = R\cos\alpha$, and the number of hops is $\geqslant d/x$. This proves the following theorem.

Theorem 2. The expected number of hops for *MFR* routing scheme (in an ad hoc network with uniform traffic and n nodes distributed uniformly at random in a square of side m so that each node has k neighbors on average) is $\geqslant d/(R\cos\alpha)$, where $d = |SD| = m\sqrt{2}/3$ is the expected distance between the source and destination, $\pi/(2k) = \alpha - \sin(2\alpha)/2$, and $R = m\sqrt{k/(\pi(n-1))}$ is the transmission radius.

Proof. Follows directly from above discussion. □

Theorem 2 leads to an estimate that has an error that does not exceed 18% for *MFR* scheme. We considered an improved estimate that is obtained by adding 0.5 to the

Table 4
Estimating hop count for *MFR* and greedy methods for 1000 nodes.

Density k	100	5	6	7	8	9	10	11	12	13	14
R	17.85	3.99	4.37	4.72	5.05	5.36	5.64	5.92	6.18	6.44	6.68
α	0.29	0.81	0.76	0.72	0.69	0.66	0.63	0.61	0.6	0.58	0.56
$d/x + 0.5$	3.26	17.69	15.39	13.78	12.58	11.63	10.87	10.24	9.71	9.25	8.85

amount d/x in theorem 2. It is considerably more accurate, with errors not exceeding 5% for *MFR* method and 7% for *greedy/GEDIR* method, as indicated in table 3.

Table 4 presents our estimates for the hop count for $n = 1000$ nodes (distributed at random 7 in a square with sides $m = 100$), and densities 5–14 and 100, for both *MFR* and greedy methods.

5. Conclusion

Our estimates are expected to be more accurate for larger number of nodes, and higher densities. The presented data are for hundred nodes and densities between 5 and 14. Note that Li [5] gave a more precise formula for the expected density, accounting for border effects. However, routing is not expected to progress in the direction of missing neighbors along the border, and therefore we believe that more precise density formula does nor reflect the routing process. However, making more precise estimates by different means may still be possible, and remains an open problem for further research. It is also of interest to estimate hop count for methods that guarantee delivery, such as *FACE* and *GFG* [Bose et al., 1]. Further research is needed to identify the best GPS based routing protocols for various network contexts. These contexts include nodes positioned in three-dimensional space and obstacles, nodes with unequal transmission powers [Stojmenovic and Lin, 8], or networks with unidirectional links. Estimating hop counts in these cases pose interesting open problems for further research.

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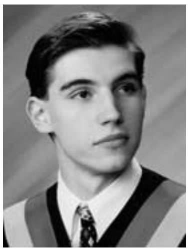

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