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Conic properties of planar point sets

Milos Stojmenović, Amiya Nayak
SITE, University of Ottawa,
Ottawa, Ontario, Canada K1N 6N5
{mstoj075, anayak}@site.uottawa.ca

Abstract— Basic shape descriptors such as linearity, circularity and ellipticity are useful tools in the field of pattern recognition. It is possible to identify groups of points as lines, circles or ellipses in an image. This is potentially a useful step in image based object detection. Our goal is to find algorithms that give shape measurements in the interval [0, 1], where values close to 1 indicate that the shape in question is a line, circle or ellipse, and values close to 0 indicate the opposite. We are interested in measures which are invariant to rotation, scaling, and translation. These measures should also be calculated very quickly and be resistant to protrusions in the data set. All of the measures surveyed here are shape boundary based which makes them applicable to point sets such as extracted edges from real world images. The main linearity measures are found in [13]. Circularity measures were discussed in [11], and an overview of such measures applied to unordered point sets is presented here. The circularity of unordered data is determined directly from the linearity measure, whereas the circularity of ordered data is derived by multiplying the unordered data circularity measure by a monotonicity factor. The proposed algorithms work on both open and closed curves. Direct ellipse fitting methods, discussed in [12], are guaranteed to specifically return an ellipse as the fit rather than any conic. The proposed algorithms work well on both open and closed curves. They are used to classify a set of 25 curves in 4 categories: line, circle, ellipse, or no shape. These three measures together could form a base for a larger shape detection system in images.

Keywords: Linearity, Circularity, Elongation, point sets.

I. INTRODUCTION

Shape analysis has maintained constant interest in the research community for the last half century. It has however fostered even more enthusiasm lately since it has become a useful and practical component of larger computer vision and inspection systems. The real world application of shape analysis and recognition can be illustrated in the following examples. Recognition of rectangular and orthonormal line segment in satellite imagery corresponds to man made structures and settlements [17]. Matching gradient features is used in handwritten character recognition [14]. Detection of ellipses, circles, sine curves and other patterns is used in finding man made objects as cups, car wheels and car outlines [15]. Sets of points which appear linear are interesting since they often represent a region of interest in an image. Most man made structures or objects have strong straight lines that are easily identifiable. By dissecting an object into an ordered collection of curves, the object becomes more easily identifiable; visually and computationally. Objects such as cars or tables in such edge representations of images are still easily recognizable by humans – the whole is more than just the sum of its parts. This leads to interesting possibilities for the domain of computer vision in the sense that useful information can be extracted from images just by examining the edges.

In this paper, we are interested in surveying measures of linearity, circularity and ellipticity. In analyzing various algorithms, we align ourselves with the following criteria. We are interested in shape measures that assign linearity, circularity and ellipticity values to sets of points. A curve's shape description should be invariant under similarity transformations, such as scaling, rotation and translation. The algorithms should also be resistant to protrusions in the data set. The shape measure values should also be computed by simple and fast algorithms. It is important to stress that points in the set are not ordered. Because the set of points is not ordered, permutations of the input set should not affect the shape measure value.

Although several linearity values have been proposed in [13]: eccentricity, contour smoothness, triangle heights, triangle perimeters, rotation correlation, average orientations, and ellipse axis ratio, we will use the most effective one: Average Orientations.

Circularity measures were discussed in [5, 2, 1, 3, 6]. All of them are area based and linked to closed curves except for one: [6]. This one is shape based and can be applied to open curves. [11] adapted the linearity measures in [13] to form circularity measures. Their linearity measures are applicable since the input set of points to the circularity algorithms of [11] was transformed from Cartesian representation to polar representation, where highly circular input point sets become highly linear in the new representation, given a proper choice of center. The linearity of the polar set corresponds to the circularity of the original Cartesian set.

Ellipse fitting has been widely studied in literature. Voss and Süße [16] described an area and moments based method of fitting geometric primitives. Rosin [7, 8] discussed various ellipse fitting methods and various distance measures of points in data sets from the corresponding ellipse fit [9]. Various ellipticity measures were proposed in literature. An ellipse fit is a prerequisite for some of them. These include DFT and the shape based measure by Proffit [6], the set operations area based method [4], and the Euclidean ellipticity shape based and orthogonal hyperbolae area based measures by Rosin [10]. Other measures are not based on a prior ellipse fit, such as the elliptic variance shape based method [6], and the area and moment based method [10]. The ellipse fit in [12] is done by first choosing a shape center and finding the orientation line of the shape. The best ellipse fit is chosen by determining the location of the foci of this fit along the orientation line in opposite directions of the shape center. The best locations of the foci are those that minimize the variance of sums of distances of points to the foci. Their ellipticity measure is based on a measure of linearity used in [13]. Since the input set of points to their algorithm was transformed from planar representation to polar representation, highly elliptical input point sets become highly linear in the new representation. Therefore, the linearity of the polar set corresponds to the circularity of the original Cartesian set.

II. Overview of Methods

We will describe several well known functions on finite sets of points that are used in our linearity measures here.

A. Discussion on Moments, Orientation and Correlation

The central moment of order *pq* of a set of points *Q* is:

$$\mu_{pq} = \frac{1}{S}\sum_{x,y\in Q}(x - x_c)^p (y - y_c)^q ,$$

where *S* is the number of points in *Q*, and (x_c, y_c) is the center of mass of the set *Q*. The center of mass is the average value of each coordinate in the set. The angle of orientation of the set of points *Q* is determined by [C]:

$$angle = 0.5\arctan\left(\frac{2\mu_{11}}{\mu_{20} - \mu_{02}}\right).$$

All definitions are applied on a set of points with real coordinates. Moments, however, are also defined for infinite sets of points, such as open or closed curves, or for all points located inside a closed curve. Moments are typically used on a closed curve, where all point within the closed curve (the area of the closed curve) are considered in their calculation. We use the moment calculations on just the finite set of points which are on the border of a closed curve, or all the points of an open curve. It was observed that the formula for the orientation line sometimes produces a line that is orthogonal to the desired one. We prove (omitted for space limits) that the actual orientation is either the one from the above formula or its orthogonal line, and used this fact in several linearity measures. We find that the orientation of the border points of a closed curve is almost identical to the orientation of all of the digital points inside the closed curve. We are especially interested in digitized curves which are used in our experiments.

B. Linearity

1) Average Orientations

Here, we first find the center of mass of the point set, and its angle of orientation using moments. This function takes *k* random pairs of points along the curve. It finds their slopes (*m*), and finds the normals to their slopes (-*m*, 1). Each normal is saved as a vector (*-m/norm*, *1/norm*) in array *ab*, where *norm*= $\sqrt{m^2 + 1}$ is a normalization factor. These vectors are compared against the normal to the orientation line determined by the moments formula above (-*M*, 1), where *M*=tan(*angle*). The dot product of (-*M*, 1) and (-*m*, 1), for each pair of points is evaluated as *dp=mM+1.* If *dp<0,* the vector (*m/norm*, *-1/norm*) is stored instead. All normals are oriented to point in the same general direction with respect to the vector (-*M*, 1). They are pointed in the same direction since the vectors would otherwise cancel each other out in the case of a perfectly straight line, and give a linearity value near 0. Please see Figure 1 below.

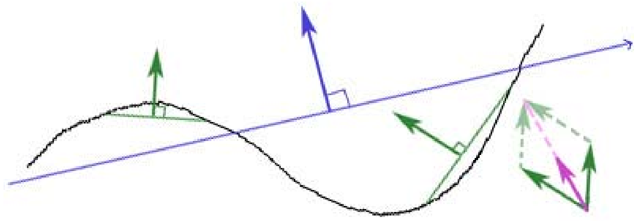

Figure 1. Normals all oriented in the same direction

These normals in array *ab* are averaged out, and the resulting normal (A, B) is deemed to be the normal to the orientation of the curve. The averaging is done separately for each vector coordinate. The measure of linearity is defined as $\sqrt{A^2 + B^2}$. In the case of a perfectly straight line, all of the unit vectors would point in the same direction, and have a height of 1 with respect to the orientation line. Otherwise, the resulting average orientation would not be orthogonal to the orientation line, and would have a magnitude less than 1. Since the moment function sometimes produces orientation lines which are 90 degrees offset from what is visually the actual orientation line, we repeat the entire procedure for $angle = angle + \pi/2$, and select the higher of the two measures.

The linearity measure produces numbers in the interval $[2/\pi, 1]$ (for a circle it is $2/\pi \approx 0.636$). This is normalized to [0,1]: $linearity \leftarrow (linearity - 2/\pi)/(1 - 2/\pi)$;

2) Rotation Correlation/Eccentricity

Correlation is a standard tool in statistics for determining whether there is a relation between two sets of points. If we consider the x and y values of points in a space separately, and apply correlation, we can directly measure linearity. Again, we first find the center of mass of the set of points along with its orientation. In this algorithm, the curve in question is rotated so that its new orientation is at an angle of 45^o from the x-axis. Correlation is then done on the rotated curve. The linearity measure is the absolute value of the measured correlation of points (x_i, y_i) on the rotated curve. The procedure is repeated for the orientation line which is at an $angle1 = angle + \pi/2$, where *angle* is the original angle of orientation as determined by the moment function. The final output of the program is the greater of the two correlation values.

Eccentricity was the simplest measure of linearity we could find. It was also used in [C]. The output of this algorithm is already in the interval [0, 1], so there was no need to normalize it. Eccentricity measures the elongation of a disc. Since lines are degenerate discs, this measure can be directly applied to measuring linearity. For a disc, this measure outputs 0, for a line, it outputs 1 since lines are eccentric. The formula is

$$linearity = \frac{\sqrt{(\mu_{20} - \mu_{02})^2 + 4\mu_{11}^2}}{\mu_{20} + \mu_{02}}.$$

[13] proves that rotation correlation and eccentricity always yield the same linearity measures.

3) Triangle heights

Here, we take k triplets of random points from the set and compute the heights h to the longest side of the triangles that the triplets form. This h value is divided by the longest side c of the triangle to normalize the measure. This value is called *hc*. We use the average of these k *hc* values as a linearity measure of the set of points. Figure 2 illustrates this point.

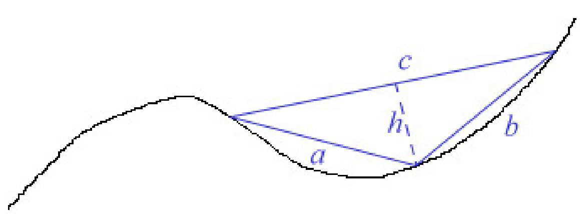

Figure 2. Triangle formed by 3 random points, and its height h

Obviously, a low average of *hc* would represent a linear set of lines. Therefore, the average *hc* value is adjusted to fit the norm of higher linearity values representing linear sets of points. The minimum value of *hc* is 0. The maximum ratio for a height of a triangle is obtained in an equilateral triangle. In such cases, $h = \sqrt{3}a/2$, where a is the length of a side of an equilateral triangle, and the ratio is $\sqrt{3}/2$. To define a measure that will allocate 1 to linear points, and 0 to the considered case of three vertices of an equilateral triangle, each hc value is adjusted as follows: $hc = 1 - \left(2hc/\sqrt{3}\right)$.

The range of obtained linearity values of this algorithm are still in the range of (0.66, 1) for the examples that we tested. The minimum value of 0.66 is obtained for circles. We stretch out this interval by adjusting the linearity as follows:

$$linearity = (hc - 0.66)/0.34 .$$

4) Triangle Perimeters

This method is similar to the previous one in the sense that we take k triplets of random points from the set of points and compute a variation of the perimeters of the triangles that the triplets form. The three sides of the triangle are labeled a, b and c, where $a \le b \le c$. The measure that we are interested in is $p = (2c-a-b)/c$. If these three points form a triangle which is degenerate in the form of a line, then p close to 1. The minimum value is 0 for the vertices of an equilateral triangle. We take the average value p to measure linearity. The linearity measure of circles is found to be 0.76. The value 0.76 was the lowest obtained p value in our experiments, and was therefore mapped to 1. Therefore, we need to stretch as follows:

$$linearity = (p - 0.76)/0.24 .$$

Although p can have values of 1 in theory (for the set of 3 points which are the vertices of an equilateral triangle), in practice the random triplets rarely produce an equilateral triangle, and experimentation shows that it is best to stretch the linearity interval using the parameters shown.

5) Contour Smoothness

The original *smoothness* formula in [C] was defined as $4\pi S/P^2$. In their formula, S is the area of the shape, and P is its perimeter. This is another measuring scheme that was adapted for linearity. It bases its measurements on the area of a shape divided by the square of its perimeter. This measure was inspired by the compactness measure. We did not take the area of the entire shape into consideration at once. Instead, we once again applied our technique of sampling the point set by taking triplets of points, and averaging out their triangular areas. Each triplet of points produces a *smoothness* value in the form of *area/perimeter*^{2}. The maximum value for area divided by the triangle perimeter is $\sqrt{3}/36$ (for an equilateral triangle). After *smoothness* values are averaged to produce value *sums*, the result is adjusted as follows: $sums = 36\ sums / \sqrt{3}$.

This limits *sums* to 1. We reversed the meaning of this *smoothness* measurement by taking the compliment of the obtained value. The measured value for circles is then 0.45. The final measure is $linearity = (1 - sums - 0.45)/0.55$.

6) Ellipse Axis Ratio

We use the idea of measuring rectangularity as proposed in [9], and adapt it to measuring linearity. The concept is similar to the eccentricity measurement. We first find the center of mass and the first and second order moments of the set of input points, and then find the values of the major and minor axis of the best fit ellipse as determined by the formulas in [9]. The linearity value is 1-*minor axis/major axis*.

C. Circularity

The circularity measures seen in [11] are presented here. The choice of center for each shape is an important factor in measuring circularity. Two methods for finding the center of each shape were used. The first method is the general center of gravity of the shape, which corresponds to a per-coordinate average value of each pixel in the shape. The true center finding method takes the median point value of k triplets sample points belonging to the curve.

The linearity measures presented in [13] were used to measure circularity here. The measures themselves were not modified. Only the input to each measure was modified. The intended input for the linearity functions was an array of n Cartesian pixel coordinates in the form (x_i, y_i). The Cartesian pixel array was transformed into an array of polar coordinate pixels. Points are transferred from Cartesian coordinates to Polar coordinates as follows: Point (x, y) in Cartesian form would be represented by $\left(\sqrt{x^2 + y^2}, \arctan(y/x)\right)$, in polar form. This form of point representation is beneficial since circular objects whose points are transferred to polar coordinates appear linear when these points are mapped. Figure 3 shows a circle with radius r drawn in planar Cartesian coordinates, and its corresponding polar representation on the right.

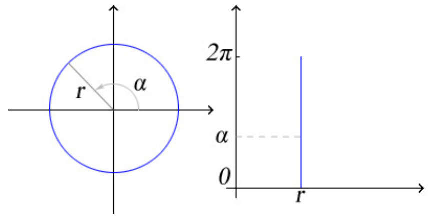

Figure 3. Cartesian and polar representations

The linearity measures of [13] were used to measure the circularity of polar coordinate input sets.

The trivial way of choosing a shape's center is to take the per-coordinate average of all pixels, which is referred to as the center of gravity. This is the method that is usually chosen when measuring any shape property such as linearity. Choosing the appropriate center of a shape when measuring circularity is more delicate and heavily influences the result of the circularity measure. Imagine a semi circular shape. Transferring the shape to polar coordinates with respect to its center of gravity would not yield a straight line, but rather a curved one. This might result in a lower circularity measure than expected for the given shape. In order to find the 'true center' of a shape, as opposed to its traditional center of gravity, k triplets of points from its point set were sampled. From each triplet, the center (X_{tc}, Y_{tc}) that the points define was found. It is expected that each triplet of points will yield a different center (X_i, Y_i). To choose a unique center for the shape, the k center values per coordinate were individually sorted, and the median per coordinate value was chosen to be the true center, (X_{tc}, Y_{tc}).

D. Ellipticity

Ellipse fitting and measuring were described in detail in [12]. Their ellipse measures are presented here in two sections. The first deals with fitting an ellipse to point data, and the second measures the ellipticity of the point set by rating the accuracy of the ellipse fit. The quality of the fit relies on an accurate method of finding the center of the shape. They propose two ways of finding the shape center. The standard way is by considering the center of gravity, and the other is by finding the true center of the shape.

1) Fitting an ellipse to a set of points

Here we describe the algorithm that fits an ellipse to a set of points. Its input is just the set of points, and it outputs the locations of the optimal foci locations, along with the major and minor axes and the angle of orientation of the major axis of the fit ellipse.

We begin by finding the angle of orientation α of the point set via moments. The moment based algorithm sometimes produces orientation angles that are normal to the actual shape orientation. To verify the correctness of the obtained value α, the linearity of the set was measured twice. The first measure was made considering the orientation angle was α, and the second was made considering orientation angle $\alpha + 90^{\circ}$. Linearity can be measured using any one of the linearity measures from [13]. The higher of the two linearity values corresponds to the actual orientation of the shape.

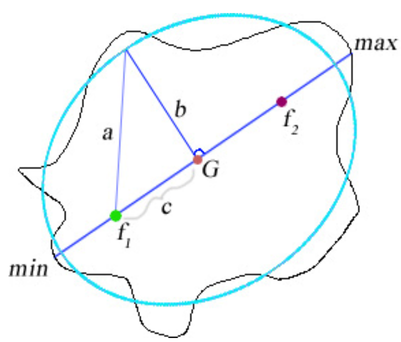

Figure 4. Orientation line with foci, min, max a, b, c, and G

The orientation line passes through the selected center and has slope α. We then project all points onto the orientation line, resulting in a new array. The two extremity points *min* and *max* along the orientation line of the new array are found. In Figure 4 we see the blue orientation line which is also the line on which the points on the shape are projected. G is the center of the shape. Foci f_1 and f_2 will be determined by the foci finding procedure to follow.

2) Finding optimal foci for ellipse fitting

For simplicity, we assume that the point set has been translated such that its center is at the origin. Also, the point set has been rotated such that its orientation line lies on the x axis.

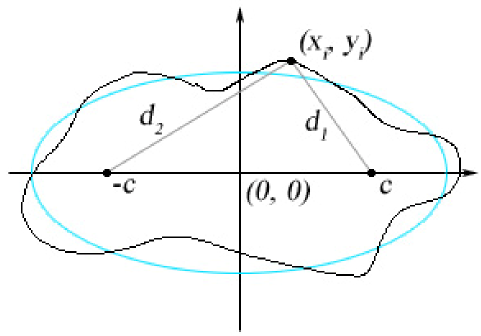

Figure 5. Varience of summed foci distances

In Figure 5, the distances to the foci are:

$$d_1 = \sqrt{(x_i - c)^2 + y_i^{\,2}} \text{, and } d_2 = \sqrt{(x_i + c)^2 + y_i^{\,2}} \text{ .}$$

Therefore, we have

$$D_i = d_1 + d_2 = \sqrt{(x_i - c)^2 + y_i^{\,2}} + \sqrt{(x_i + c)^2 + y_i^{\,2}} \text{ .}$$

We need to find c for which the values D_i have the smallest possible variance. Variance is defined by:

$$f(c) = (N-1)\sigma^2 = \sum_{i=1}^{N} D_i^{\,2} - \frac{1}{N}\left(\sum_{i=1}^{N} D_i\right)^2 .$$

This is a continuous function and thus has a minimum value c such that $f'(c) = 0$. It can be solved by a standard equation root finding technique of numerical analysis, such as the bisection method. Note that, $f(c)$ may have local minima and thus multiple solutions for equation $f'(c) = 0$, some of which could even correspond to local maxima. Therefore solving the problem in this direction is not straight forward. [12] therefore opted for a simple approximate solution that corresponds to a linear search with pixel unit distance steps.

Now that the foci are found, we need the median of the sum of distances from each point on the shape to both foci in order to find the length of the major axis a, which equals half of this sum. The distance c from focus f_1 to center G is used to find the length of the minor axis $b = \sqrt{a^2 - c^2}$. Now all of the necessary components of the ellipse fit are available in order to be able to evaluate it.

3) Assesing the fit quality: minimal variance of summed foci distances

Once the foci of the ellipse fit have been determined, the quality of the ellipse fit can be assessed. This was done by first transforming the original point set into polar representation. This transformation is seen in Figure 6.

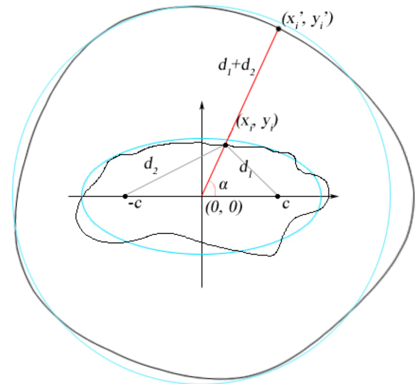

Figure 6. Transforming the input set to polar representation

In Figure 6, the inherent ellipse property that each point on the ellipse is equidistant to the sum of distances from both foci is seen. [12] exploit this property when transforming the point set to polar coordinate form. As seen in the bottom part of Figure 6, the polar distance value for each point x from the center G will be the sum of distances from x to both foci, $r=d_1+d_2$. The angle α that vector Gx' forms with the x-axis will remain the same as the angle Gx formed with the x-axis. For a perfect ellipse, the resulting shape can be drawn as a circle, but if its polar coordinate shapes are plotted as Cartesian, they would look highly linear, as seen in Figure 7.

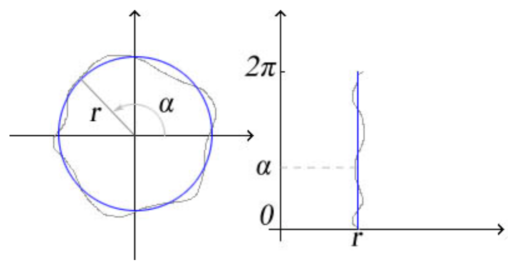

Figure 7. Polar point set on a planar graph

Applying a linearity measure to this polar representation results in a linearity value for the modified set of points. This linearity value represents the ellipticity value of the original set.

A normalization was applied to the polar coordinate transformation of points since the angles the points form in the new representation is limited to the interval [0, 360], whereas the length of the radii they form is unbounded. This can result in polar representations that are not proportional to the original shape. To normalize the polar representation, the following information is gathered from it: the length of the smallest and largest radii r_{min}, r_{max}, and the variance of the radii, r_{var}. The normalization factor $norm = (360*r_{var})/N$.

This normalization factor represents the size of the interval each radius would be fit into. The larger the variance of the radii, the less the points fit to the ellipse. Therefore the interval they are placed in is larger to make the linearity value of this set smaller. Each radius value is normalized by the following statement: $R=norm*(r-r_{min})/(r_{max}-r_{min})$. The result is a number that fits a radius R into the interval [0, $norm$].

4) Average distance ratio to ellipse and the center

[12] propose a shape based ellipticity measure that can be applied to open and closed shapes. Let O be the center of fitted ellipse, and V be a point from the original set. Let U be the intersection of line VO with the fitted ellipse that is closer to V, therefore $|VU|<|VO|$. This intersection can be found by using the equations of a line and ellipse, which leads to a quadratic equation. The error measure used is the average of the distance ration to the ellipse and to the center, that is, $|VU|/|VO|$. It always returns a number between 0 and 1. Please see Figure 8 for clarification.

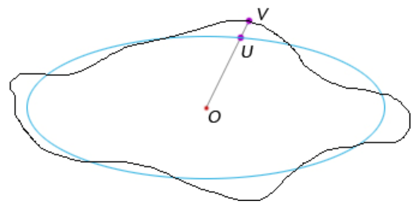

Figure 8. Finding intersect *U* on line *VO*

E. 3.4 Finding the center of a shape

The trivial way of choosing a shape's center is to take the per-coordinate average of all pixels, which is the center of gravity. This is the method that is usually chosen when measuring any shape property such as linearity, orientability or elongation. Choosing the appropriate center of a shape when measuring ellipticity is more delicate and heavily influences the result of the ellipticity measure. To illustrate this point, please see figure 9.

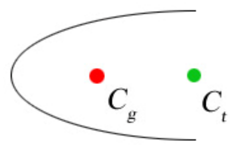

Figure 9. Choosing the correct shape center

Here, we see a semi elliptical shape where the red dot represents the center of the shape as determined by the traditional method. Transferring the shape to polar coordinates with respect to the red dot would not yield a straight line, but rather a curved one. This would result in a much lower ellipticity measure than expected for the given shape. However, had the green dot been chosen as the center, the resulting polar coordinate representation would have looked similar to the line seen in Figure 4, and a much higher ellipticity measure would have been awarded.

[12] experimented with both types of center finding methods in this work. In order to find the 'true center' of a shape, as opposed to its traditional center of gravity, they sampled *k* quintuplets of points from its point set. From each quintuplet, they found the center (X_{tc}, Y_{tc}) that the points define via the method proposed in [7]. It was however found that the center of gravity method generally yields better results for the majority of curves.

III. EXPERIMENTS AND DISCUSSION

To illustrate the abilities of the linearity, circularity and ellipticity measures seen here, we have devised the following experiment. Figure 10 shows a list of 25 boundary based shapes, mostly taken from [13]. All three of the linearity [13], circularity [11], and ellipticity [12], algorithms are evaluated on each shape. Each shape is deemed to be closest to a basic shape if its basic shape measure is the highest of the three. In this fashion, each test shape will be declared as being close or perfectly equivalent to a line, a circle or an ellipse.

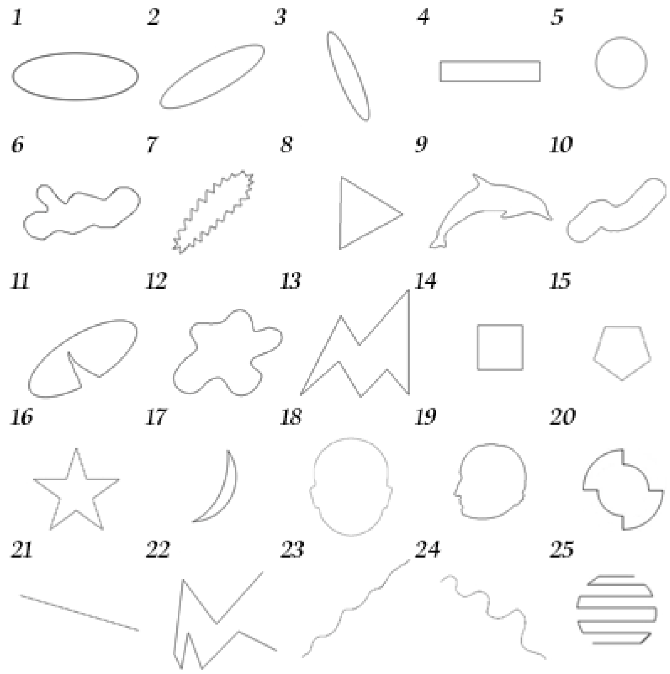

Figure 10. Test shapes for linearity, circularity, and ellipticity

Table I shows the shape measurements of the curves in Figure 10. The linearity, circularity, and ellipticity measures are in the [13], [11] and [12] columns respectively. Each of these measures assumes an unordered set of input points, and understands that each shape's centroid is its center of gravity. The Average Orientations (AO) method of linearity was used in the [13] column to measure linearity. The circularity method is from [11] and use the AO method for finding the linearity of the polar coordinate set. Ellipticity was measured via the method described above. These three methods were chosen since they most accurately measure the basic shape properties we are analyzing. The 'shape' column specifies which of the three measures best suits the given shape, and classifies each shape as either line, circle, or ellipse. The '80% certainty' column introduces a threshold which deems a shape unrecognizable as any of the three basic shapes chosen if none of their basic shape measures is at least 0.8. Shapes below that threshold are labeled 'no shape' since they do not closely match any of the three given categories.

TABLE I. SHAPE MEASURES

	[13]	**[11]**	**[12]**	**shape**	**%80 certainty**
1	0.460	0.783	0.988	ellipse	ellipse
2	0.520	0.525	0.981	ellipse	ellipse
3	0.595	0.554	0.979	ellipse	ellipse
4	0.632	0.576	0.901	ellipse	ellipse
5	0.017	1.000	0.992	circle	circle
6	0.461	0.562	0.827	ellipse	ellipse
7	0.538	0.480	0.896	ellipse	ellipse
8	0.039	0.804	0.824	ellipse	ellipse
9	0.388	0.444	0.709	ellipse	no shape
10	0.502	0.509	0.849	ellipse	ellipse
11	0.445	0.552	0.860	ellipse	ellipse
12	0.181	0.741	0.825	ellipse	ellipse
13	0.171	0.515	0.745	ellipse	no shape
14	0.021	0.927	0.919	circle	circle
15	0.055	0.950	0.946	circle	circle

16	0.037	0.650	0.773	ellipse	no shape
17	0.685	0.388	0.680	line	no shape
18	0.106	0.930	0.978	ellipse	ellipse
19	0.081	0.963	0.958	circle	circle
20	0.175	0.778	0.928	ellipse	ellipse
21	0.989	0.083	0.745	line	line
22	0.091	0.558	0.689	ellipse	no shape
23	0.965	0.077	0.751	line	line
24	0.744	0.143	0.736	line	no shape
25	0.022	0.744	0.731	circle	no shape

We conclude that the shapes above were correctly classified as lines, circles or ellipses. We notice that a circle is a specific case of an ellipse , and as such shape 5 is highly quoted as both a circle and an ellipse. Shape 21 is clearly a line, but a line is also a degenerate ellipse. Its rating was highest as a line, and it was classified as such, but its ellipticity value was not very far behind. Shapes 9, 13, 16, 17, 22, 24, 25 were rightly rejected from the 80% category. Shape 25 looks like a circle, but it has far too many contours within its circular shape to qualify as a clear circle.

Using the methods outlined above it is possible to classify curves as lines, circles and ellipses. A more detailed study into this field would take into consideration real world images, and extract actual curves from them. Identifying these curves as basic shapes would prove very beneficial to object detection in computer vision systems.

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